Modified Element Pattern Overlap Integrals for Antenna Array Mutual Impedance Estimation

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***Abstract*—From the conservation of energy principle, the real part of the mutual impedance between antenna elements, or the mutual resistance, can be found for lossless antennas by computing embedded element pattern overlap integrals. Using the asymptotic behavior of mutual impedances for large separation distances, mutual reactance can be estimated from the derivative of the pattern overlap integrals with respect to element separation. This allows the pattern overlap integral to be modified to provide an estimate of the reactive part of the mutual coupling, so that pattern overlap integrals give the complex mutual impedance between antennas including both real and imaginary parts. For numerical methods that compute overlap integrals to estimate array thermal noise response, gain, or beam-dependent efficiencies, a simple modification can be made to the algorithm so that the overlap integrals provide mutual impedances. Numerical results show that the modified overlap integral method for mutual impedance estimation is reasonably accurate for dipole antennas in the near and far fields.**

I. Introduction

Determining the mutual coupling between antenna elements in an array is needed for a variety of applications, including multiple input multiple output (MIMO) communications, arrays for magnetic resonance imaging, and high-sensitivity astronomical imaging systems. Even with fast numerical methods, for large imaging arrays, full wave numerical simulation of array coupling can require days to weeks. In these applications, fast and accurate methods for rapidly estimating mutual coupling are desirable.

One method for approximating mutual impedances is the induced electromotive force (EMF) method, which has been known since the 1950s [1]. More sophisticated techniques with greater accuracy are available that offer greater accuracy and can be valid in the near field [2]–[5], but these methods are generally difficult to implement in comparison to classical techniques for coupling estimation.

Another approach to determining mutual coupling is based on integrals of inner products of embedded element patterns (EEPs), or pattern overlap integrals. Overlap integrals have been referred to as beam coupling factors [6]. Pattern overlap integrals can be used to find the response of an array to external thermal noise, antenna gain for coupled arrays, aperture efficiency for formed beams, and other antenna parameters and figures of merit [7]–[9]. Using conservation of energy, for a lossless array the real part of the mutual impedance matrix can be found from array element pattern overlap integrals [7], [9], [10]. This is an exact relationship that holds in the near and far fields. Pattern overlap integrals in their standard form, however, only give the real mutual coupling and not the reactive part.

Here we use the asymptotic behavior of mutual impedances as a function of separation distances to develop a modified pattern overlap integral that gives both the real and reactive parts of the mutual coupling between elements in an antenna array. With this method, existing overlap integral codes can be modified in a simple way to compute complex mutual impedances for antenna arrays from the element patterns.

II. Derivation

A. Mutual coupling in the far field

Using reciprocity and the far field approximation, the mutual coupling between two elements in an array is the open circuit voltage at the terminals of the mth element in an array induced by the field radiated by element n, which can be approximated in the far field as [9, Eq. (2.38)]

$$Z_{mn} \simeq \frac{4\pi j r e^{jkr}}{\omega\mu I_0}\overline{E}_m \cdot \overline{E}_n(\overline{r}) \tag{1}$$

Since this formula is based on the reciprocity principle, the dot product is computed without a complex conjugate. $\overline{E}_n$ is the field radiated by the nth element evaluated at the origin of the coordinate system used to compute the far field radiated by the mth element. $\overline{E}_m(\overline{r})$ is evaluated at a point $\overline{r}$ in the direction of the nth element's phase center and $r = |\overline{r}|$. This approximation for the coupling between antennas is related to the induced EMF method [1] for finding the coupling between antennas.

The leading dependence of the incident field $\overline{E}_n$ on the distance d from the mth element in the far field is

$$\overline{E}_n(\overline{r}) \sim \frac{e^{-jkd}}{d}, \quad r \to \infty \tag{2}$$

The r dependence of $\overline{E}_m$ in (1) is removed by the factor re^{jkr}. The mutual impedance has the same dependence on distance between elements as the far field of an antenna, so that

$$Z_{mn} \sim \frac{e^{-jkd}}{d}, \quad r \to \infty \tag{3}$$

for large distances between elements m and n. By evaluating the derivative with respect to distance d, it can be seen that to first order,

$$\frac{\partial}{\partial d} Z_{mn} \simeq -jkZ_{mn} \tag{4}$$

The real part of this equation is

$$\frac{\partial}{\partial d}\,\mathrm{Re}[Z_{mn}] \simeq k\,\mathrm{Im}[Z_{mn}] \tag{5}$$

This means that the real and imaginary parts of the mutual impedance are in quadrature with respect to d and we can approximate the reactive part of the mutual impedance from the real part by

$$\mathrm{Im}[Z_{mn}] \simeq \frac{1}{k}\frac{\partial}{\partial d}\mathrm{Re}[Z_{mn}] \tag{6}$$

This relationship will provide a way to estimate reactive coupling from antenna array pattern overlap integrals.

B. Modified pattern overlap integrals

For a lossless antenna array, the real part of the mutual impedance matrix is given by the pattern overlap integrals [9]

$$\mathrm{Re}[Z_{mn}] = \frac{1}{\eta|I_0|^2}\oint \overline{E}_m(\overline{r})\cdot\overline{E}_n^*(\overline{r})\,d\overline{r} \tag{7}$$

where the region of integration is a surface enclosing the antenna array and $\overline{E}_m$ is the embedded element pattern with the mth element excited by input current I_0 and the other elements terminated with open circuit loads. EEPs are found by exciting one element in the array at a time. Since EEPs depend on the loads or terminations on the nondriven elements, the loading condition must be considered when using (7). If the EEPs are simulated or measured for, say, a matched loading condition, a matrix transformation can be used to convert the EEPs to the open circuit loading condition [11].

If the array elements are approximated as identical other than a shift in location, then the element patterns are related by

$$\overline{E}_n(\overline{r}) \simeq \overline{E}_m(\overline{r})e^{jk\hat{r}\cdot\overline{r}_{mn}} \tag{8}$$

where $\overline{r}_{mn} = \overline{r}_n - \overline{r}_m$ is the relative position vector between elements m and n. Inserting this into the overlap integral gives

$$\mathrm{Re}[Z_{mn}] = \frac{1}{\eta|I_0|^2}\oint \left|\overline{E}_m(\overline{r})\right|^2 e^{jk\hat{r}\cdot\overline{r}_{mn}}\,d\overline{r} \tag{9}$$

Using (6), the reactive part of the mutual impedance can be approximated as

$$\mathrm{Im}[Z_{mn}] \simeq \frac{1}{\eta|I_0|^2}\frac{1}{k}\frac{\partial}{\partial d}\oint \left|\overline{E}_m(\overline{r})\right|^2 e^{jkd\hat{r}\cdot\hat{r}_{mn}} \tag{10}$$

where $d = r_{mn}$. Evaluating the derivative leads to

$$\mathrm{Im}[Z_{mn}] \simeq \frac{1}{\eta|I_0|^2}\oint |\overline{E}_m(\overline{r})|^2 j\hat{r}\cdot\hat{r}_{mn}e^{jk\hat{r}\cdot\overline{r}_{mn}} \tag{11}$$

Combining this expression with (7) and removing the approximation (8) gives the modified pattern overlap integral

$$Z_{mn} \simeq \frac{1}{\eta|I_0|^2}\oint \overline{E}_m(\overline{r})\cdot\overline{E}_n^*(\overline{r})(1-\hat{r}\cdot\hat{r}_{mn})\,d\overline{r} \tag{12}$$

for the mutual impedance between elements m and n. This expression is similar to (7), except that it has the additional term $-\hat{r}\cdot\hat{r}_{mn}$ in the integrand and gives both the real and reactive coupling between the elements.

Aside from the simple computation required to find the dot product of the unit vector corresponding to the integration point with the unit offset vector $\hat{r}_{mn}$ between the two elements, the integral in (12) requires no more cost than evaluating the standard array element pattern overlap integral, yet it provides the exact real coupling between the elements and an estimate of the reactive coupling.

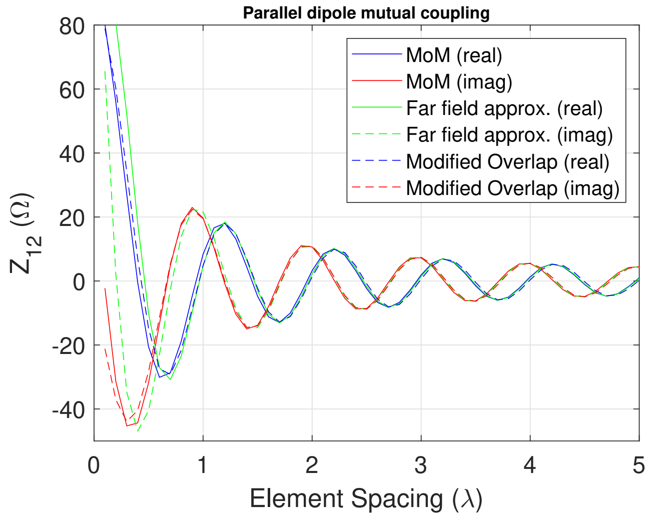

Fig. 1. Mutual impedance for parallel half wave dipoles as a function of separation distance. For the parallel case and separation distances larger than a wavelength, the modified overlap integral (12) is similar in value to the far field induced EMF approximation (1). For closer separation distances, the modified overlap integral is more accurate.

III. Results

The accuracy of the modified overlap integral approximation for the mutual coupling between antenna elements was studied by comparing to numerical results for half wave dipoles in parallel and collinear configurations.

Figure 1 shows the case of parallel dipoles. The method of moments was used to generate reference values. The modified overlap integral (12) gives similar results to the far field induced EMF method in (1) and both are close to the method of moments results for element spacings larger than a wavelength. For elements spaced more closely than a wavelength, the modified overlap integral values are closer to the method of moments than the far field approximation.

Results for collinear dipoles are shown in Fig. 2. In this case, the induced EMF method gives a zero value for the mutual impedance. The modified overlap integral results agree reasonably well with the method of moments even for spacings between the collinear elements that are small enough that the dipoles are nearly connected.

IV. Conclusion

We have developed a modified overlap integral that gives the exact mutual resistance and an estimate of the mutual reactance. Although the result was derived using the far field approximation, since the traditional overlap integral method for finding mutual resistance, ignoring antenna loss, is exact and holds in both the near and far fields, one might expect that the modified overlap integral is reasonably accurate in the near field as well as the far field. Numerical results suggest that this may be the case.

One benefit of the proposed method is that modeling codes that already compute pattern overlap integrals for modeling the response of the array to isotropic thermal noise [9] or

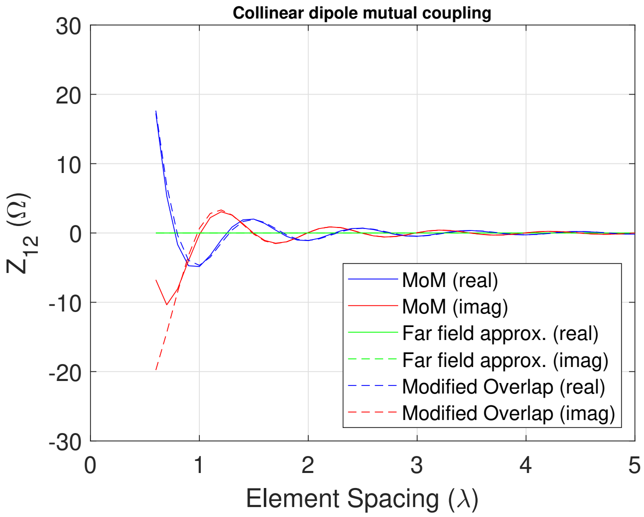

Fig. 2. Mutual impedance for collinear half wave dipoles. The far field approximation (1) is zero, as the array elements radiate no field in the direction of the dipole axis. The modified overlap integral (12) is reasonably accurate even for relatively closely spaced elements.

other purposes can be modified in a simple way to provide the mutual impedance including both the real and imaginary parts.

Another benefit of the proposed method is that it can be used to estimate mutual coupling between elements in different subarrays in a way that includes the effect of neighboring elements on the coupling between distant elements. For a large array with two disconnected subarrays, a full wave numerical method can be used to find the embedded element patterns for the elements in the first subarray. In a separate computation the EEPs are found for the second subarray. If the modified overlap integral in (12) is used to find the coupling between an element in the first subarray and an element in the second subarray, the coupling estimate includes the effects of neighboring elements or structures near either element in the two subarrays. In view of the computational cost scaling of full wave numerical methods, modeling the two subarrays separately is significantly faster than modeling the full array, particularly when the arrays are large in relation to the operating wavelength.

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